

Superfield Approach to Symmetries for Matter Fields in Abelian Gauge Theories

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Abstract: The derivation of the nilpotent Becchi-Rouet-Stora-Tyutin (BRST)- and anti-BRST symmetries for the matter fields, present in any arbitrary interacting gauge theory, has been a long-standing problem in the framework of superfield approach to BRST formalism. These nilpotent (anti-)BRST symmetries for the Dirac fields are derived in the superfield formulation for the interacting Abelian gauge theory in four $(3 + 1)$ -dimensions (4D) of spacetime. The same type of symmetries are deduced for the 4D complex scalar fields having a gauge invariant interaction with the $U(1)$ gauge field. The above interacting theories are considered on a six $(4 + 2)$ -dimensional supermanifold parametrized by four *even* spacetime coordinates and a couple of *odd* elements of the Grassmann algebra. The invariance of the conserved matter (super)currents and the horizontality condition on the (super)manifolds play very important roles in the above derivations. The geometrical origin and interpretation for all the above off-shell nilpotent symmetries are provided in the framework of superfield formalism.

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1 Introduction

The Becchi-Rouet-Stora-Tyutin (BRST) formalism is one of the most elegant methods for the covariant canonical quantization of the gauge theories as well as the reparametrization invariant theories which are endowed with the first-class constraints in the language of the Dirac's prescription for the classification of constraints [1,2]. In the realm of frontier areas of research connected with topological field theories [3-5] and (super)string theories (see, eg, [6,7] and references therein), the reach and range of the applicability of BRST formalism are overwhelming. The scope of this formalism has been beautifully extended to encompass the second-class constraints in its domain of applications [8]. Its geometrical interpretation in the framework of superfield formulation and its intimate connections with the basic tenets of supersymmetry [9-14], its mathematically consistent inclusion in the Batalin-Vilkovisky formalism [15,16], its deep connections with the basic ideas behind the differential geometry and cohomology [17-21], etc., have elevated the subject of BRST formalism to a high degree of mathematical sophistication and very useful physical applications. The true strength of the BRST formalism appears in its full glory in the context of interacting non-Abelian gauge theories where the unitarity and "quantum" gauge (i.e. BRST) invariance are respected together at any arbitrary order of perturbative computation for a given physical process involving the matter fields and the non-Abelian gauge fields (which are clearly the true physical fields of the theory). In this context, it is pertinent to point out that for each loop diagram consisting of the (gluon) gauge fields, there exists a corresponding loop diagram consisting of the (anti-)ghost fields (which are not the physical fields of the theory in the true sense) so that the unitarity can be maintained for a given physical process (see, eg, [22] for details).

In our present endeavour, we shall be concentrating *only* on the key points associated with the geometrical aspects of the superfield approach applied to BRST formalism. This superfield technique is one of the most interesting and intuitive approaches to gain an insight into the geometrical meaning of the conserved and nilpotent ($Q_{(a)b}^2 = 0$) (anti-)BRST charges as well as the nilpotent ($s_{(a)b}^2 = 0$) (anti-)BRST symmetries they generate for the Lagrangian density of a given p -form ($p = 1, 2, 3, \dots$) gauge theory defined on the D -dimensional spacetime manifold. The key idea in this formulation is to consider the D -dimensional p -form gauge theory on a $(D + 2)$ -dimensional supermanifold parametrized by the D -number of spacetime (even) coordinates x^μ ($\mu = 0, 1, 2, 3, \dots, D - 1$) and a couple of Grassmannian (odd) variables θ and $\bar{\theta}$ (with $\theta^2 = \bar{\theta}^2 = 0, \theta\bar{\theta} + \bar{\theta}\theta = 0$). One constructs the super curvature $(p + 1)$ -form $\tilde{F} = \tilde{d}\tilde{A} + \tilde{A} \wedge \tilde{A}$ from the super exterior derivative \tilde{d} (with $\tilde{d}^2 = 0$) and the super connection p -form \tilde{A} . This is finally equated, due to the so-called *horizontality condition*, with the ordinary curvature $(p + 1)$ -form $F = dA + A \wedge A$ constructed from the ordinary exterior derivative $d = dx^\mu \partial_\mu$ (with $d^2 = 0$) and the p -form ordinary connection A . The above restriction is referred to as the *soul-flatness* condition in [23] which amounts to setting equal to zero all the Grassmannian components of the $(p + 1)$ -

rank (anti-)symmetric curvature tensor that is required in the definition of the $(p+1)$ -form super curvature on the $(D+2)$ -dimensional supermanifold. The procedure of reducing the $(D+2)$ -dimensional super curvature \tilde{F} to the D -dimensional ordinary curvature F in the horizontality restriction ($\tilde{F} = F$) leads to (i) the derivation of the nilpotent (anti-)BRST symmetry transformations on the gauge field and the (anti-)ghost fields of the p -form gauge theory, (ii) the geometrical interpretation for the (anti-)BRST charges $Q_{(a)b}$ as the translation generators along the Grassmannian directions of the supermanifold, and (iii) the geometrical meaning of the nilpotency ($s_{(a)b}^2 = 0, Q_{(a)b}^2 = 0$) property which is found to be encoded in a couple of successive translations (i.e. $(\partial/\partial\theta)^2 = (\partial/\partial\bar{\theta})^2 = 0$) along any particular Grassmannian direction (i.e. θ or $\bar{\theta}$) of the supermanifold.

Recently, in a set of papers [24-26], all the three super cohomological operators ($\tilde{d}, \tilde{\delta}, \tilde{\Delta}$) corresponding to the ordinary de Rham cohomological operators [†] have been exploited in the generalized versions of the horizontality condition defined on a four $(2+2)$ -dimensional supermanifold to demonstrate the existence of the local, covariant and continuous (anti-)BRST-, (anti-)co-BRST- and a bosonic symmetry (which is equal to the anticommutator(s) of the (anti-)BRST and (anti-)co-BRST symmetries) transformations for the two $(1+1)$ -dimensional (2D) free Abelian gauge theory. The above symmetry transformations have also been discussed in the canonical Lagrangian formulation of this theory [27-29]. Exactly similar kind of symmetry transformations for the self-interacting 2D non-Abelian gauge theories have also been obtained in the Lagrangian formulation [30] as well as superfield formulation [31]. The topological nature of the above 2D (non-)Abelian gauge theories has also been captured in the superfield formulation where, for the first time, the geometrical origin for the Lagrangian density and the symmetric energy momentum tensor has been provided. In fact, these physical quantities have been shown to correspond to the translations of some local (but composite) superfields along the Grassmannian directions of the supermanifold [32,33]. In a very recent paper [34], the local, covariant, continuous and nilpotent (anti-)BRST symmetry transformations and the non-local, non-covariant, continuous and nilpotent (anti-)co-BRST transformations have been shown to exist in the superfield formulation for the 4D interacting Abelian gauge theory defined on the six $(4+2)$ -dimensional supermanifold. For the 4D free Abelian 2-form gauge theory, the local, covariant, continuous and nilpotent (anti-)BRST and (anti-)co-BRST symmetries as well as a bosonic symmetry have also been obtained in the Lagrangian formulation [35,36].

In all the above-cited papers on the superfield formalism, only the nilpotent transformations for the gauge field and the (anti-)ghost fields have been obtained by exploiting the (dual-)horizontality conditions on the supermanifolds (see, eg, [26,34] for detail references). The horizontality condition ($\tilde{F} = F$) and the dual-horizontality condition ($\tilde{d}\tilde{A} = \delta A$) owe

[†]The set (d, δ, Δ) of operators, defined on a compact manifold without a boundary, is called the set of de Rham cohomological operators where $\delta = \pm * d*$, $d = dx^\mu \partial_\mu$, $\Delta = (d + \delta)^2$ are called the (co-)exterior derivatives $((\delta)d)$ and the Laplacian operator (Δ) respectively. Here $*$ is the Hodge duality operation on the manifold. These operators obey an algebra: $d^2 = \delta^2 = 0$, $\Delta = \{d, \delta\}$, $[\delta, \Delta] = 0$, $[d, \Delta] = 0$ showing that the Laplacian operator Δ is the Casimir operator for the whole algebra (see, eg, [17,18] for details).

their origin to (i) the super (co-)exterior derivatives $(\tilde{\delta})\tilde{d}$ and their ordinary counterparts $(\delta)d$, and (ii) the super 1-form connection \tilde{A} and its ordinary counterpart A . In physical terms, the above conditions originate due to the gauge (or BRST) invariance of the $(p+1)$ -form (super)curvatures $(\tilde{F})F$ and the dual-gauge (or co-BRST) invariance of the (super) zero-forms $\tilde{\delta}\tilde{A}$ and δA , respectively. It is obvious that, in the above conditions on the (super)manifolds, the matter fields of the interacting gauge theory *do not* play any role at all. As a consequence, these conditions do not shed any light on the derivation of the nilpotent symmetry transformations for the matter fields of the theory. To the best of our knowledge, in the known literature on the superfield formulations [9-14, 24-26], there has been no definite clue on the derivation of the nilpotent symmetry transformations for the matter fields. This is why, it has been a long-standing problem to derive the nilpotent transformations for the matter fields for an *interacting* gauge theory in any arbitrary dimension of spacetime. In this connection, it is worthwhile to mention that, in a very recent paper [37], it has been shown that the invariance of the conserved matter (super)currents on the four $(2+2)$ -dimensional (super)manifold leads to the derivation of the nilpotent (anti-)BRST transformations for the Dirac fields in an interacting 2D Abelian gauge theory where the matter conserved current $J_\mu^{(d)} = \bar{\psi}\gamma_\mu\psi$ couples to the $U(1)$ gauge field A_μ . For the massless Dirac fields, it has been shown that the invariance of the (super) axial-vector current, constructed by the (super) matter fields, leads to the derivation of the local, covariant, continuous and off-shell nilpotent (anti-)co-BRST transformations on the massless Dirac fields.

The purpose of the present paper is to demonstrate that the invariance of the vector conserved (super)currents (i.e. $\tilde{J}_\mu^{(d,c)}(x, \theta, \bar{\theta}) = J_\mu^{(d,c)}(x)$), constructed by the (super) Dirac and (super) complex scalar fields, on the six $(4+2)$ -dimensional supermanifold leads to the derivation of the off-shell nilpotent, local, covariant and continuous (anti-)BRST symmetries for the Dirac- as well as complex scalar fields. We would like to lay emphasis on the fact that, the requirement of the invariance of the matter (super)currents, is not a restriction put by hand from outside. Rather, it is the inherent and innate feature of the interacting gauge theory itself. Thus, this condition emerges automatically, unlike the case of the (dual-)horizontal conditions (see, eg, [26] for details) which are imposed by hand on the supermanifold. For the case of the interacting $U(1)$ gauge theory with the Dirac fields, we show that the horizontality condition does not play any significant role in the derivation of the nilpotent (anti-)BRST symmetries for the Dirac fields. This is because of the fact that the matter conserved current $J_\mu^{(d)} = \bar{\psi}\gamma_\mu\psi$ does not contain, in any way, the other physical field A_μ of the theory. On the contrary, in the case of the interacting Abelian gauge theory involving the complex scalar fields, the horizontality condition does play a very important role in the derivation of the nilpotent symmetries on the matter (complex scalar) fields. The root cause of this crucial role, played by the horizontality condition, is the presence of the gauge field A_μ in the conserved matter current $J_\mu^{(c)} \sim \phi^*\partial_\mu\phi - \phi\partial_\mu\phi^* + 2ieA_\mu\phi^*\phi$ constructed by the complex scalar fields. In fact, the

interacting $U(1)$ gauge theory with the complex scalar field provides a really interesting physical system where the horizontality condition and the invariance of the conserved matter (super)currents on the (super)manifolds are found to be consistent with each-other. This mutual consistency entails upon the nilpotent transformations for the gauge field, the (anti-)ghost fields and the matter fields to be complementary to one-another. We comment more on this *consistency issue* in the conclusion (cf section 6) part of our present paper.

The contents of our present paper are organized as follows. In section 2, we give a brief synopsis of the off-shell nilpotent (anti-)BRST symmetries for the interacting $U(1)$ gauge theory in the Lagrangian formulation where the gauge field A_μ is coupled to the conserved matter currents constructed by (i) the Dirac fields, and (ii) the complex scalar fields. For the sake of this paper to be self-contained, section 3 deals with the derivation of the above nilpotent symmetries for the gauge- and (anti-)ghost fields in the framework of superfield formulation where the horizontality condition on the six $(4+2)$ -dimensional supermanifold plays a crucial role [12,26]. The central theme of our paper is contained in sections 4 and 5 where we derive the off-shell nilpotent symmetries for the Dirac and complex scalar fields, respectively, by exploiting the invariance of the conserved matter (super)currents on the (super)manifolds. We lay emphasis on our key results, make some concluding remarks and point out a few future directions for further investigations in section 6.

2 Nilpotent (anti-)BRST symmetries: Lagrangian formulation

To recapitulate the key points connected with the local, covariant, continuous and off-shell nilpotent (anti-)BRST symmetries for the Lagrangian density \mathcal{L}_b of an *interacting* four $(3+1)$ -dimensional (4D) $U(1)$ gauge theory [‡] in the Feynman gauge, we begin with [38-40]

$$\begin{aligned}\mathcal{L}_b &= -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \bar{\psi} (i\gamma^\mu D_\mu - m) \psi + B (\partial \cdot A) + \frac{1}{2} B^2 - i \partial_\mu \bar{C} \partial^\mu C \\ &\equiv \frac{1}{2} (\mathbf{E}^2 - \mathbf{B}^2) + \bar{\psi} (i\gamma^\mu D_\mu - m) \psi + B (\partial \cdot A) + \frac{1}{2} B^2 - i \partial_\mu \bar{C} \partial^\mu C\end{aligned}\quad (2.1)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength tensor for the $U(1)$ gauge field A_μ that is derived from the 2-form $dA = \frac{1}{2}(dx^\mu \wedge dx^\nu)F_{\mu\nu}$. As is evident, the latter is constructed by the application of the exterior derivative $d = dx^\mu \partial_\mu$ (with $d^2 = 0$) on the 1-form $A = dx^\mu A_\mu$ (which defines the vector potential A_μ). The gauge-fixing term $(\partial \cdot A)$ is derived through the operation of the co-exterior derivative δ (with $\delta = - * d *$, $\delta^2 = 0$) on the one-form A (i.e. $\delta A = - * d * A = (\partial \cdot A)$) where $*$ is the Hodge duality operation. The fermionic Dirac fields $(\psi, \bar{\psi})$, with mass m and charge e , couple to the $U(1)$ gauge field A_μ (i.e. $-e\bar{\psi}\gamma^\mu A_\mu\psi$) through the conserved current $J_\mu^{(d)} = \bar{\psi}\gamma_\mu\psi$. The anticommuting

[‡]We adopt here the conventions and notations such that the 4D flat Minkowski metric is: $\eta_{\mu\nu} = \text{diag}(+1, -1, -1, -1)$ and $\square = \eta^{\mu\nu} \partial_\mu \partial_\nu = (\partial_0)^2 - (\partial_i)^2$, $F_{0i} = \partial_0 A_i - \partial_i A_0 = E_i \equiv \mathbf{E}$, $F_{ij} = \epsilon_{ijk} B_k$, $B_i \equiv \mathbf{B} = \frac{1}{2}\epsilon_{ijk} F_{jk}$, $(\partial \cdot A) = \partial_0 A_0 - \partial_i A_i$, $D_\mu \psi = \partial_\mu \psi + ie A_\mu \psi$ where \mathbf{E} and \mathbf{B} are the electric and magnetic fields, respectively. The totally antisymmetric Levi-Civita tensor $\varepsilon_{\mu\nu\lambda\xi}$ and the 4×4 Dirac γ -matrices are chosen to satisfy: $\varepsilon_{0123} = -\varepsilon^{0123} = +1$, $\varepsilon_{0ijk} = \epsilon_{ijk}$, $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$ where ϵ_{ijk} is the Levi-Civita tensor in the space submanifold. Here the Greek indices: $\mu, \nu, \lambda, \dots = 0, 1, 2, 3$ correspond to the spacetime directions and Latin indices $i, j, k, \dots = 1, 2, 3$ stand only for the space directions on the manifold.

($C\bar{C} + \bar{C}C = 0, C^2 = \bar{C}^2 = 0, C\psi + \psi C = 0$ etc.) (anti-)ghost fields (\bar{C}) C are required to maintain both the unitarity and “quantum” gauge (i.e. BRST) invariance together at any arbitrary order of perturbation theory [§]. The Nakanishi-Lautrup auxiliary field $B = -(\partial \cdot A)$ is required to linearize the usual gauge-fixing term $-\frac{1}{2}(\partial \cdot A)^2$ of the theory. Under the following off-shell nilpotent ($s_{(a)b}^2 = 0$) (anti-)BRST symmetry transformations $s_{(a)b}$ [¶] on the gauge-, (anti-)ghost- and matter fields (with $s_b s_{ab} + s_{ab} s_b = 0$) [38-40]

$$\begin{aligned} s_b A_\mu &= \partial_\mu C & s_b C &= 0 & s_b \bar{C} &= iB & s_b \psi &= -ieC\psi \\ s_b \bar{\psi} &= -ie\bar{\psi}C & s_b \mathbf{B} &= 0 & s_b B &= 0 & s_b \mathbf{E} &= 0 & s_b(\partial \cdot A) &= \square C \\ s_{ab} A_\mu &= \partial_\mu \bar{C} & s_{ab} \bar{C} &= 0 & s_{ab} C &= -iB & s_{ab} \psi &= -ie\bar{C}\psi \\ s_{ab} \bar{\psi} &= -ie\bar{\psi}\bar{C} & s_{ab} \mathbf{B} &= 0 & s_{ab} B &= 0 & s_{ab} \mathbf{E} &= 0 & s_{ab}(\partial \cdot A) &= \square \bar{C} \end{aligned} \quad (2.2)$$

the above Lagrangian density transforms to a total derivative. The above transformations are generated by the off-shell nilpotent ($Q_{(a)b}^2 = 0$) and conserved (anti-)BRST charges $Q_{(a)b}$.

The other dynamically closed system ^{||} that respects the above kind of symmetry transformations is the system of complex scalar fields coupled to the $U(1)$ gauge field A_μ . This system is described by the following Lagrangian density (see, eg, [41])

$$\begin{aligned} \mathcal{L}_B &= -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + (D_\mu \phi)^* D^\mu \phi - V(\phi^* \phi) + B(\partial \cdot A) + \frac{1}{2} B^2 - i \partial_\mu \bar{C} \partial^\mu C \\ &\equiv \frac{1}{2} (\mathbf{E}^2 - \mathbf{B}^2) + (D_\mu \phi)^* D^\mu \phi - V(\phi^* \phi) + B(\partial \cdot A) + \frac{1}{2} B^2 - i \partial_\mu \bar{C} \partial^\mu C \end{aligned} \quad (2.3)$$

where $V(\phi^* \phi)$ ^{**} is the potential describing the interaction between the complex scalar fields ϕ and ϕ^* and the covariant derivatives on these fields are

$$D_\mu \phi = \partial_\mu \phi + ieA_\mu \phi \quad (D_\mu \phi)^* = \partial_\mu \phi^* - ieA_\mu \phi^*. \quad (2.4)$$

It will be noted that the gauge field A_μ couples to the conserved matter current $J_\mu^{(c)} \sim [\phi^* D_\mu \phi - \phi (D_\mu \phi)^*]$ to provide the interaction between the $U(1)$ gauge field and matter fields ϕ and ϕ^* (cf (2.3)). This statement can be succinctly expressed by re-expressing (2.3), in terms of the kinetic energy terms for ϕ and ϕ^* , as

$$\begin{aligned} \mathcal{L}_B &= -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \partial_\mu \phi^* \partial^\mu \phi - ieA_\mu [\phi^* \partial_\mu \phi - \phi \partial_\mu \phi^*] + e^2 A^2 \phi^* \phi \\ &\quad - V(\phi^* \phi) + B(\partial \cdot A) + \frac{1}{2} B^2 - i \partial_\mu \bar{C} \partial^\mu C. \end{aligned} \quad (2.5)$$

[§]The full strength of the (anti-)ghost fields is realized in the discussion of the unitarity and gauge invariance for the perturbative computations in the realm of non-Abelian gauge theory where the Feynman graphs involve the loop diagrams of the gauge (gluon) fields (see, eg, [22] for details). In fact, to maintain the unitarity, there exists a ghost loop diagram corresponding to a loop diagram involving only the gauge field. This is required to counter the contributions coming out from the gauge loop graph [22].

[¶]We adopt here the notations and conventions followed by Weinberg [40]. In its totality, the nilpotent ($\delta_B^2 = 0$) BRST transformation δ_B is the product of an anticommuting (i.e. $\eta C + C\eta = 0, \eta\psi + \psi\eta = 0$ etc.) spacetime independent parameter η and s_b as $\delta_B = \eta s_b$ where $s_b^2 = 0$.

^{||}In the sense of the basic requirements of a canonical field theory, the Lagrangian density \mathcal{L}_B (cf (2.3)) describes a dynamically closed system because the quadratic kinetic energy terms and the interaction terms for all the fields ϕ, ϕ^* and A_μ are present in this Lagrangian density in a logical fashion [41].

^{**}For a renormalizable quantum field theory, this potential can be chosen in the quartic polynomial form as: $V(\phi^* \phi) = \mu^2 \phi^* \phi + \lambda (\phi^* \phi)^2$ where μ and λ are the parameters which could be chosen in different ways for different purposes. For instance, the free field theory corresponds to $\lambda = 0, \mu^2 > 0$ (see, eg, [41] for details). The key point to be noted here is the fact that $V(\phi^* \phi)$ remains invariant under (2.6).

The conservation ($\partial \cdot J^{(c)} = 0$) of the matter current $J_\mu^{(c)}$ (which couples to the gauge field A_μ in the above Lagrangian density) can be easily checked by exploiting the equations of motion $D_\mu D^\mu \phi = -(\partial V / \partial \phi^*)$, $(D_\mu D^\mu \phi)^* = -(\partial V / \partial \phi)$ derived from the Lagrangian densities (2.3) and/or (2.5). The above Lagrangian density respects the following off-shell nilpotent (anti-)BRST transformations on the matter fields, gauge field and the (anti-)ghost fields:

$$\begin{aligned}
s_b A_\mu &= \partial_\mu C & s_b C &= 0 & s_b \bar{C} &= iB & s_b \phi &= -ieC\phi \\
s_b \phi^* &= +ie\phi^* C & s_b \mathbf{B} &= 0 & s_b B &= 0 & s_b \mathbf{E} &= 0 & s_b (\partial \cdot A) &= \square C \\
s_{ab} A_\mu &= \partial_\mu \bar{C} & s_{ab} \bar{C} &= 0 & s_{ab} C &= -iB & s_{ab} \phi &= -ie\bar{C}\phi \\
s_{ab} \phi^* &= +ie\phi^* \bar{C} & s_{ab} \mathbf{B} &= 0 & s_{ab} B &= 0 & s_{ab} \mathbf{E} &= 0 & s_{ab} (\partial \cdot A) &= \square \bar{C}.
\end{aligned} \tag{2.6}$$

The key points to be noted, at this stage, are (i) under the (anti-)BRST transformations, it is the kinetic energy term $-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$ that remains invariant. This statement is true for any gauge theory. For the above $U(1)$ gauge theory, as it turns out, it is the curvature term $F_{\mu\nu}$ (constructed from the operation of the exterior derivative d on the 1-form $A = dx^\mu A_\mu$) itself that remains invariant under the (anti-)BRST transformations. (ii) In the mathematical language, the (anti-)BRST symmetries owe their origin to the exterior derivative d because the curvature term is constructed from it. (iii) The gauge- and (anti-)ghost fields are endowed with exactly the same symmetry transformations for both the cases of the interacting Abelian gauge theories that are being considered here (cf (2.2) and (2.6)). (iv) In general, the above transformations can be concisely expressed in terms of the generic field $\Sigma(x)$ and conserved charges $Q_{(a)b}$, as

$$s_r \Sigma(x) = -i [\Sigma(x), Q_r]_\pm \quad r = b, ab \tag{2.7}$$

where the local generic field $\Sigma = A_\mu, C, \bar{C}, \psi, \bar{\psi}, B, \phi, \phi^*$ and the (+)– signs, as the subscripts on the (anti-)commutators $[\ ,]_\pm$, stand for Σ being (fermionic)bosonic in nature.

3 The gauge- and (anti-)ghost fields: nilpotent symmetries

We exploit here the superfield formalism to obtain the off-shell nilpotent symmetry transformations for A_μ, C, \bar{C} fields present in (2.2) and/or (2.6). To this end in mind, we begin with a six $(4+2)$ -dimensional supermanifold parametrized by the general superspace coordinate $Z^M = (x^\mu, \theta, \bar{\theta})$ where x^μ ($\mu = 0, 1, 2, 3$) are the four even spacetime coordinates and $\theta, \bar{\theta}$ are a couple of odd elements of a Grassmann algebra. On this supermanifold, one can define a supervector superfield $\tilde{A}_M = (B_\mu(x, \theta, \bar{\theta}), \mathcal{F}(x, \theta, \bar{\theta}), \bar{\mathcal{F}}(x, \theta, \bar{\theta}))$ with $B_\mu, \mathcal{F}, \bar{\mathcal{F}}$ as the component multiplet superfields [12,11]. These multiplet superfields can be expanded in terms of the basic fields A_μ, C, \bar{C} and the secondary fields as (see, eg, [11,12,26])

$$\begin{aligned}
B_\mu(x, \theta, \bar{\theta}) &= A_\mu(x) + \theta \bar{R}_\mu(x) + \bar{\theta} R_\mu(x) + i \theta \bar{\theta} S_\mu(x) \\
\mathcal{F}(x, \theta, \bar{\theta}) &= C(x) + i \theta \bar{B}(x) + i \bar{\theta} B(x) + i \theta \bar{\theta} s(x) \\
\bar{\mathcal{F}}(x, \theta, \bar{\theta}) &= \bar{C}(x) + i \theta \bar{B}(x) + i \bar{\theta} B(x) + i \theta \bar{\theta} \bar{s}(x).
\end{aligned} \tag{3.1}$$

It is straightforward to note that the local fields $R_\mu(x), \bar{R}_\mu(x), C(x), \bar{C}(x), s(x), \bar{s}(x)$ are fermionic (anti-commuting) in nature and the bosonic (commuting) local fields in (3.1) are: $A_\mu(x), S_\mu(x), \mathcal{B}(x), \bar{\mathcal{B}}(x), B(x), \bar{B}(x)$. It is clear that, in the above expansion, the bosonic- and fermionic degrees of freedom match and, in the limit $\theta, \bar{\theta} \rightarrow 0$, we get back our basic fields A_μ, C, \bar{C} of (2.1) and/or (2.5). These requirements are essential for the sanctity of any arbitrary supersymmetric theory in the superfield formulation. In fact, all the secondary fields will be expressed in terms of basic fields due to the restrictions emerging from the application of horizontality condition (i.e. $\tilde{F} = F$), namely;

$$\tilde{F} = \frac{1}{2} (dZ^M \wedge dZ^N) \tilde{F}_{MN} = \tilde{d}\tilde{A} \equiv dA = \frac{1}{2}(dx^\mu \wedge dx^\nu) F_{\mu\nu} = F \quad (3.2)$$

where the super exterior derivative \tilde{d} and the connection super one-form \tilde{A} are defined as

$$\begin{aligned} \tilde{d} &= dZ^M \partial_M = dx^\mu \partial_\mu + d\theta \partial_\theta + d\bar{\theta} \partial_{\bar{\theta}} \\ \tilde{A} &= dZ^M \tilde{A}_M = dx^\mu B_\mu(x, \theta, \bar{\theta}) + d\theta \tilde{\mathcal{F}}(x, \theta, \bar{\theta}) + d\bar{\theta} \mathcal{F}(x, \theta, \bar{\theta}). \end{aligned} \quad (3.3)$$

In physical language, the requirement (3.2) implies that the physical field \mathbf{E} and \mathbf{B} , derived from the curvature term $F_{\mu\nu}$, do not get any contribution from the Grassmannian variables. In other words, the physical electric field \mathbf{E} and magnetic field \mathbf{B} for the 4D QED remain intact in the superfield formulation. Mathematically, the condition (3.2) implies the “flatness” of all the components of the super curvature (2-form) tensor \tilde{F}_{MN} that are directed along the θ and/or $\bar{\theta}$ directions of the supermanifold. To this end in mind, first we expand $\tilde{d}\tilde{A} = \tilde{F}$ (for the interacting Abelian gauge theory under consideration) as

$$\begin{aligned} \tilde{d}\tilde{A} &= (dx^\mu \wedge dx^\nu) (\partial_\mu B_\nu) - (d\theta \wedge d\theta) (\partial_\theta \tilde{\mathcal{F}}) + (dx^\mu \wedge d\bar{\theta}) (\partial_\mu \mathcal{F} - \partial_{\bar{\theta}} B_\mu) \\ &- (d\theta \wedge d\bar{\theta}) (\partial_\theta \mathcal{F} + \partial_{\bar{\theta}} \tilde{\mathcal{F}}) + (dx^\mu \wedge d\theta) (\partial_\mu \tilde{\mathcal{F}} - \partial_\theta B_\mu) - (d\bar{\theta} \wedge d\bar{\theta}) (\partial_{\bar{\theta}} \mathcal{F}). \end{aligned} \quad (3.4)$$

Ultimately, the application of soul-flatness (horizontality) condition ($\tilde{d}\tilde{A} = dA$) yields [26]

$$\begin{aligned} R_\mu(x) &= \partial_\mu C(x) & \bar{R}_\mu(x) &= \partial_\mu \bar{C}(x) & s(x) &= \bar{s}(x) = 0 \\ S_\mu(x) &= \partial_\mu B(x) & B(x) + \bar{B}(x) &= 0 & \mathcal{B}(x) &= \bar{\mathcal{B}}(x) = 0. \end{aligned} \quad (3.5)$$

The insertion of all the above values in the expansion (3.1) yields

$$\begin{aligned} B_\mu(x, \theta, \bar{\theta}) &= A_\mu(x) + \theta \partial_\mu \bar{C}(x) + \bar{\theta} \partial_\mu C(x) + i \theta \bar{\theta} \partial_\mu B(x) \\ \mathcal{F}(x, \theta, \bar{\theta}) &= C(x) - i \theta B(x) & \tilde{\mathcal{F}}(x, \theta, \bar{\theta}) &= \bar{C}(x) + i \bar{\theta} B(x). \end{aligned} \quad (3.6)$$

This equation leads to the derivation of the (anti-)BRST symmetries for the gauge- and (anti-)ghost fields of the Abelian gauge theory (cf (2.2) and (2.6)). In addition, this exercise provides the physical interpretation for the (anti-)BRST charges $Q_{(a)b}$ as the generators (cf eqn. (2.7)) of translations (i.e. $\text{Lim}_{\bar{\theta} \rightarrow 0}(\partial/\partial\theta), \text{Lim}_{\theta \rightarrow 0}(\partial/\partial\bar{\theta})$) along the Grassmannian directions of the supermanifold. Both these observations can be succinctly expressed, in a combined fashion, by re-writing the super expansion (3.1) as

$$\begin{aligned} B_\mu(x, \theta, \bar{\theta}) &= A_\mu(x) + \theta (s_{ab} A_\mu(x)) + \bar{\theta} (s_b A_\mu(x)) + \theta \bar{\theta} (s_b s_{ab} A_\mu(x)) \\ \mathcal{F}(x, \theta, \bar{\theta}) &= C(x) + \theta (s_{ab} C(x)) + \bar{\theta} (s_b C(x)) + \theta \bar{\theta} (s_b s_{ab} C(x)) \\ \tilde{\mathcal{F}}(x, \theta, \bar{\theta}) &= \bar{C}(x) + \theta (s_{ab} \bar{C}(x)) + \bar{\theta} (s_b \bar{C}(x)) + \theta \bar{\theta} (s_b s_{ab} \bar{C}(x)). \end{aligned} \quad (3.7)$$

A closer look at the (anti-)BRST transformations shows that the (anti-)ghost fields transform only under one of these transformations. That is to say the fact that the anti-ghost field \bar{C} transforms under the BRST transformation but it remains unchanged under the anti-BRST transformation. Exactly the opposite happens with the ghost field C . This statement can be expressed in a more sophisticated language of the conditions on the superfields. It is clear from (3.6) that the horizontality condition enforces the superfields $(\bar{\mathcal{F}}(x, \theta, \bar{\theta}))\mathcal{F}(x, \theta, \bar{\theta})$ to become (anti-)chiral due to the equivalence between the translation generators operating on superfields of the supermanifold and the internal symmetry generators $Q_{(a)b}$ acting on the local fields of the ordinary manifold (cf (4.10) below).

4 The Dirac fields: nilpotent symmetries

It is clear that, for the derivation of the off-shell nilpotent symmetries on the gauge- and (anti-)ghost fields, we do require the horizontality restriction on the supermanifold. For this purpose, the basic physical and mathematical objects we exploit, are the connection (super) one-form $(\tilde{A})A$ and the (super) exterior derivative $(\tilde{d})d$ to obtain the symmetry transformations on the above fields (by the restriction $\tilde{F} = F$). Consistent with the above nilpotent transformations, the nilpotent transformations on the Dirac fields are derived from the requirement of the invariance of the matter conserved (super)currents on the (super)manifolds. To corroborate this assertion, we begin with the super expansions for the Dirac superfields $\Psi(x, \theta, \bar{\theta})$ and $\bar{\Psi}(x, \theta, \bar{\theta})$ in terms of the basic Dirac fields $\psi(x)$ and $\bar{\psi}(x)$ and some extra secondary fields as

$$\begin{aligned}\Psi(x, \theta, \bar{\theta}) &= \psi(x) + i\theta \bar{b}_1(x) + i\bar{\theta} b_2(x) + i\theta \bar{\theta} f(x) \\ \bar{\Psi}(x, \theta, \bar{\theta}) &= \bar{\psi}(x) + i\theta \bar{b}_2(x) + i\bar{\theta} b_1(x) + i\theta \bar{\theta} \bar{f}(x).\end{aligned}\tag{4.1}$$

It is evident that, in the limit $(\theta, \bar{\theta}) \rightarrow 0$, we get back the Dirac fields $(\psi, \bar{\psi})$ of the Lagrangian density (2.1). Furthermore, the number of bosonic fields $(b_1, \bar{b}_1, b_2, \bar{b}_2)$ match with the fermionic fields $(\psi, \bar{\psi}, f, \bar{f})$ so that the above expansion is consistent with the basic tenets of supersymmetry. Now one can construct the supercurrent $\tilde{J}_\mu(x, \theta, \bar{\theta})$ from the above superfields with the following general super expansion

$$\begin{aligned}\tilde{J}_\mu^{(d)}(x, \theta, \bar{\theta}) &= \bar{\Psi}(x, \theta, \bar{\theta}) \gamma_\mu \Psi(x, \theta, \bar{\theta}) \\ &= J_\mu^{(d)}(x) + \theta \bar{K}_\mu^{(d)}(x) + \bar{\theta} K_\mu^{(d)}(x) + i\theta \bar{\theta} L_\mu^{(d)}(x)\end{aligned}\tag{4.2}$$

where the above components (i.e. $\bar{K}_\mu^{(d)}, K_\mu^{(d)}, L_\mu^{(d)}, J_\mu^{(d)}$), along the Grassmannian directions θ and $\bar{\theta}$ as well as the bosonic directions $\theta\bar{\theta}$ and identity $\hat{1}$ of the supermanifold, can be expressed in terms of the components of the basic super expansions (4.1), as

$$\begin{aligned}\bar{K}_\mu^{(d)}(x) &= i(\bar{b}_2 \gamma_\mu \psi - \bar{\psi} \gamma_\mu \bar{b}_1) & K_\mu^{(d)}(x) &= i(b_1 \gamma_\mu \psi - \bar{\psi} \gamma_\mu b_2) \\ L_\mu^{(d)}(x) &= \bar{f} \gamma_\mu \psi + \bar{\psi} \gamma_\mu f + i(\bar{b}_2 \gamma_\mu b_2 - b_1 \gamma_\mu \bar{b}_1) & J_\mu^{(d)}(x) &= \bar{\psi} \gamma_\mu \psi.\end{aligned}\tag{4.3}$$

To be consistent with our earlier observation that the (anti-)BRST transformations $(s_{(a)b})$ are equivalent to the translations $(\text{Lim}_{\bar{\theta} \rightarrow 0}(\partial/\partial\theta)) \text{Lim}_{\theta \rightarrow 0}(\partial/\partial\bar{\theta})$ along the $(\theta)\bar{\theta}$ -directions

of the supermanifold, it is straightforward to re-express the expansion in (4.2) as follows

$$\tilde{J}_\mu^{(d)}(x, \theta, \bar{\theta}) = J_\mu^{(d)}(x) + \theta (s_{ab} J_\mu^{(d)}(x)) + \bar{\theta} (s_b J_\mu^{(d)}(x)) + \theta \bar{\theta} (s_b s_{ab} J_\mu^{(d)}(x)). \quad (4.4)$$

It can be checked that, under the (anti-)BRST transformations (2.2), the conserved current $J_\mu^{(d)}(x)$ remains invariant (i.e. $s_b J_\mu^{(d)}(x) = s_{ab} J_\mu^{(d)}(x) = 0$). This statement, with the help of (4.2) and (4.4), can be mathematically expressed as

$$\begin{aligned} s_b J_\mu^{(d)} = 0 &\Rightarrow K_\mu^{(d)} = 0 \Rightarrow b_1 \gamma_\mu \psi = \bar{\psi} \gamma_\mu b_2 \\ s_{ab} J_\mu^{(d)} = 0 &\Rightarrow \bar{K}_\mu^{(d)} = 0 \Rightarrow \bar{b}_2 \gamma_\mu \psi = \bar{\psi} \gamma_\mu \bar{b}_1 \\ s_b \bar{s}_{ab} J_\mu^{(d)} = 0 &\Rightarrow L_\mu^{(d)} = 0 \Rightarrow \bar{f} \gamma_\mu \psi + \bar{\psi} \gamma_\mu f = i(b_1 \gamma_\mu \bar{b}_1 - \bar{b}_2 \gamma_\mu b_2). \end{aligned} \quad (4.5)$$

One of the possible solutions of the above restrictions, in terms of the components of the basic expansions in (4.1) and the basic fields of the Lagrangian density (2.1), is

$$\begin{aligned} b_1 &= -e\bar{\psi}C & b_2 &= -eC\psi & \bar{b}_1 &= -e\bar{C}\psi & \bar{b}_2 &= -e\bar{\psi}\bar{C} \\ f &= -ie [B + e\bar{C}C] \psi & \bar{f} &= +ie \bar{\psi} [B + eC\bar{C}]. \end{aligned} \quad (4.6)$$

At the moment, it appears to us that the above solutions are the *unique* solutions to all the restrictions in (4.5) ^{††}. However, we do not have a mathematically rigorous proof for the same. Ultimately, the restriction that emerges on the $(2+2)$ -dimensional supermanifold is

$$\tilde{J}_\mu^{(d)}(x, \theta, \bar{\theta}) = J_\mu^{(d)}(x). \quad (4.7)$$

Physically, the above mathematical equation implies that there is no superspace contribution to the ordinary conserved current $J_\mu^{(d)}(x)$. In other words, the transformations on the Dirac fields ψ and $\bar{\psi}$ (cf (2.2)) are such that the supercurrent $\tilde{J}_\mu^{(d)}(x, \theta, \bar{\theta})$ becomes a local composite field $J_\mu^{(d)}(x) = (\bar{\psi} \gamma_\mu \psi)(x)$ *vis-à-vis* equation (4.4) and there is no Grassmannian contribution to it. In a more sophisticated language, the conservation law $\partial \cdot J^{(d)} = 0$ remains intact despite our discussions connected with the superspace and supersymmetry. It is straightforward to check that the substitution of (4.6) into (4.1) leads to the following

$$\begin{aligned} \Psi(x, \theta, \bar{\theta}) &= \psi(x) + \theta (s_{ab} \psi(x)) + \bar{\theta} (s_b \psi(x)) + \theta \bar{\theta} (s_b s_{ab} \psi(x)) \\ \bar{\Psi}(x, \theta, \bar{\theta}) &= \bar{\psi}(x) + \theta (s_{ab} \bar{\psi}(x)) + \bar{\theta} (s_b \bar{\psi}(x)) + \theta \bar{\theta} (s_b s_{ab} \bar{\psi}(x)). \end{aligned} \quad (4.8)$$

This establishes the fact that the nilpotent (anti-)BRST charges $Q_{(a)b}$ are the translations generators ($\text{Lim}_{\bar{\theta} \rightarrow 0}(\partial/\partial\theta)$) $\text{Lim}_{\theta \rightarrow 0}(\partial/\partial\bar{\theta})$ along the $(\theta)\bar{\theta}$ -directions of the supermanifold. The property of the nilpotency (i.e. $Q_{(a)b}^2 = 0$) is encoded in the two successive translations along the Grassmannian directions of the supermanifold (i.e. $(\partial/\partial\theta)^2 = (\partial/\partial\bar{\theta})^2 = 0$). In a more sophisticated mathematical language, the above statement for the (anti-)BRST

^{††}Let us concentrate on $b_1 \gamma_\mu \psi = \bar{\psi} \gamma_\mu b_2$. A closer look at it makes it evident that the pair of bosonic components b_1 and b_2 should be proportional to the pair of fermionic fields $\bar{\psi}$ and ψ , respectively. To make the latter pair bosonic in nature, we have to include the ghost field C of the Lagrangian density (2.1) to obtain: $b_1 \sim \bar{\psi}C, b_2 \sim C\psi$. Rest of the choices in (4.6) follow exactly similar kind of arguments.

charges $Q_{(a)b}$ can be succinctly expressed, using (2.7), as

$$\begin{aligned} s_b \Sigma(x) &= \text{Lim}_{\theta \rightarrow 0} \frac{\partial}{\partial \theta} \tilde{\Sigma}(x, \theta, \bar{\theta}) \equiv -i \{\Sigma(x), Q_b\} \\ s_{ab} \Sigma(x) &= \text{Lim}_{\bar{\theta} \rightarrow 0} \frac{\partial}{\partial \theta} \tilde{\Sigma}(x, \theta, \bar{\theta}) \equiv -i \{\Sigma(x), Q_{ab}\} \end{aligned} \quad (4.9)$$

where the generic local field $\Sigma(x) = \psi(x), \bar{\psi}(x)$ and the generic superfield $\tilde{\Sigma}(x, \theta, \bar{\theta}) = \Psi(x, \theta, \bar{\theta}), \bar{\Psi}(x, \theta, \bar{\theta})$. Thus, it is evident that the nilpotent symmetry transformations, the corresponding nilpotent charges and the translations generators on the supermanifold are inter-related through the following mappings

$$s_b \leftrightarrow Q_b \leftrightarrow \text{Lim}_{\theta \rightarrow 0} \frac{\partial}{\partial \theta} \quad s_{ab} \leftrightarrow Q_{ab} \leftrightarrow \text{Lim}_{\bar{\theta} \rightarrow 0} \frac{\partial}{\partial \theta}. \quad (4.10)$$

The above relationship demonstrates that (i) the internal symmetry transformations on the ordinary fields, (ii) the nilpotent generators for the internal symmetry transformations, and (iii) the translation generators for the superfields on the supermanifold are inextricably intertwined with one-another.

5 The complex scalar fields: nilpotent symmetries

The central claim of our present investigation is connected with our observation that the *invariance* of the (super)currents, constructed by the (super) matter fields, on the (super)manifolds leads to the derivation of the local, covariant, continuous and off-shell nilpotent (anti-)BRST symmetry transformations for the matter fields. In the previous section, we checked the validity of the above claim in the context of the interacting $U(1)$ gauge theory where the Dirac fields were coupled to the $U(1)$ gauge field A_μ . In this context, it is crucial to note that both the conditions (i.e. horizontality restriction- and the invariance of the conserved currents on the supermanifold) are not connected with each-other in the case of interacting gauge theory with Dirac fields. These conditions are disjoint and decoupled in some sense. This is why, in the present section, we study the complex scalar field coupled to the $U(1)$ gauge field which provides an *interacting* system where the interplay between both the above restrictions plays a crucial and decisive role in the derivation of the off-shell nilpotent symmetries for the matter fields. To bolster up this statement, we start off with the super expansion of the superfields $\Phi(x, \theta, \bar{\theta})$ and $\Phi^*(x, \theta, \bar{\theta})$ in terms of the basic fields $\phi(x)$ and $\phi^*(x)$ and some extra secondary fields, as

$$\begin{aligned} \Phi(x, \theta, \bar{\theta}) &= \phi(x) + i \theta \bar{f}_1(x) + i \bar{\theta} f_2(x) + i \theta \bar{\theta} b(x) \\ \Phi^*(x, \theta, \bar{\theta}) &= \phi^*(x) + i \theta \bar{f}_2^*(x) + i \bar{\theta} f_1^*(x) + i \theta \bar{\theta} b^*(x) \end{aligned} \quad (5.1)$$

where the number of fermionic local fields $\bar{f}_1(x), f_1^*(x), f_2(x), \bar{f}_2^*(x)$ match with the number of bosonic local fields $\phi(x), \phi^*(x), b(x), b^*(x)$ to maintain the basic requirements of a supersymmetric field theory. It is obvious that, in the limit $(\theta, \bar{\theta}) \rightarrow 0$, we retrieve our starting

basic complex scalar fields ϕ and ϕ^* . In terms of the above superfields, we can write the expression for the supercurrent on the supermanifold as

$$\begin{aligned} \tilde{J}_\mu^{(c)}(x, \theta, \bar{\theta}) &= \Phi^*(x, \theta, \bar{\theta}) \partial_\mu \Phi(x, \theta, \bar{\theta}) - \Phi(x, \theta, \bar{\theta}) \partial_\mu \Phi^*(x, \theta, \bar{\theta}) \\ &+ 2 i e B_\mu(x, \theta, \bar{\theta}) \Phi(x, \theta, \bar{\theta}) \Phi^*(x, \theta, \bar{\theta}) \end{aligned} \quad (5.2)$$

where $B_\mu(x, \theta, \bar{\theta})$ is the superfield corresponding to the vector $U(1)$ gauge field $A_\mu(x)$ that has the expansion (3.6). It will be recalled that this expansion is obtained after the application of the horizontality condition. The above supercurrent can be expanded, in general, along the $\hat{\mathbf{1}}, \theta, \bar{\theta}$ and $\theta\bar{\theta}$ -directions of the supermanifold as

$$\tilde{J}_\mu^{(c)}(x, \theta, \bar{\theta}) = J_\mu^{(c)}(x) + \theta \bar{K}_\mu^{(c)}(x) + \bar{\theta} K_\mu^{(c)}(x) + i \theta \bar{\theta} L_\mu^{(c)}(x) \quad (5.3)$$

where the individual components on the r.h.s can be expressed as follows

$$\begin{aligned} J_\mu^{(c)}(x) &= \phi^* \partial_\mu \phi - \phi \partial_\mu \phi^* + 2ie A_\mu \phi^* \phi & L_\mu^{(c)}(x) &= L_{\mu 1}^{(c)} + L_{\mu 2}^{(c)} \\ K_\mu^{(c)} &= i [\phi^* \partial_\mu f_2 + f_1^* \partial_\mu \phi - (\partial_\mu \phi^*) f_2 - (\partial_\mu f_1^*) \phi] \\ &- 2e [A_\mu (\phi^* f_2 + f_1^* \phi) - i (\partial_\mu C) \phi^* \phi] \\ \bar{K}_\mu^{(c)} &= i [\phi^* \partial_\mu \bar{f}_1 + \bar{f}_2^* \partial_\mu \phi - (\partial_\mu \phi^*) \bar{f}_1 - (\partial_\mu \bar{f}_2^*) \phi] \\ &- 2e [A_\mu (\phi^* \bar{f}_1 + \bar{f}_2^* \phi) - i (\partial_\mu \bar{C}) (\phi^* \phi)]. \end{aligned} \quad (5.4)$$

The explicit expression for $L_{\mu 1}^{(c)}$ and $L_{\mu 2}^{(c)}$, in the above equation, are

$$\begin{aligned} L_{\mu 1}^{(c)} &= i [\phi^* \partial_\mu b + b^* \partial_\mu \phi + i (f_1^* \partial_\mu \bar{f}_1 - \bar{f}_2^* \partial_\mu f_2) \\ &- (\partial_\mu \phi^*) b - (\partial_\mu b^*) \phi + i \{ (\partial_\mu \bar{f}_2^*) f_2 - (\partial_\mu f_1^*) \bar{f}_1 \}] \\ L_{\mu 2}^{(c)} &= -2 e [(A_\mu) (\phi^* b + b^* \phi + i f_1^* \bar{f}_1 - i \bar{f}_2^* f_2) - (\partial_\mu \bar{C}) (\phi^* f_2 + f_1^* \phi) \\ &- (\partial_\mu C) (\phi^* \bar{f}_1 + \bar{f}_2^* \phi) + (\partial_\mu B) (\phi^* \phi)]. \end{aligned} \quad (5.5)$$

In sections 3 and 4, we have been able to show that the nilpotent ($Q_{(a)b}^2 = 0$) (anti-)BRST charges $Q_{(a)b}$ that generate the nilpotent ($s_{(a)b}^2 = 0$) transformations correspond to the translation generators ($\lim_{\bar{\theta} \rightarrow 0} \partial/\partial\theta$) $\lim_{\theta \rightarrow 0} (\partial/\partial\bar{\theta})$ along the Grassmannian $(\theta)\bar{\theta}$ -directions of the supermanifold. This statement is valid for the derivation of the nilpotent symmetry transformations for the gauge, (anti-)ghost and matter fields of any given interacting gauge theory in the framework of the *augmented* superfield formalism. We christen our present superfield formalism, where the horizontality condition and the invariance of the matter (super)currents on the (super)manifolds are exploited together, as the *augmented* superfield formalism. To maintain the sanctity of this geometrical interpretation for the case of any arbitrary fields (eg, the composite fields $\tilde{J}_\mu^{(c)}$), it is straightforward to re-express the most general expansion (5.3) as

$$\tilde{J}_\mu^{(c)}(x, \theta, \bar{\theta}) = J_\mu^{(c)}(x) + \theta (s_{ab} J_\mu^{(c)}(x)) + \bar{\theta} (s_b J_\mu^{(c)}(x)) + \theta \bar{\theta} (s_b s_{ab} J_\mu^{(c)}(x)). \quad (5.6)$$

It can be readily verified that $s_{(a)b} J_\mu^{(c)} = 0$ where the conserved ordinary matter current $J_\mu^{(c)}(x) \sim \phi^* D_\mu \phi - \phi D_\mu \phi^*$ (cf section 2) and $s_{(a)b}$ are the off-shell nilpotent (anti-)BRST

transformations in (2.6). Insertions of these explicit values (ie $s_b J_\mu^{(c)} = 0, s_{ab} J_\mu^{(c)} = 0$) in (5.6) imply the natural equality of the super matter current and the ordinary matter current (ie $\tilde{J}_\mu^{(c)}(x, \theta, \bar{\theta}) = J_\mu^{(c)}(x)$) because all the individual terms on the rhs of (5.6) vanish. The comparison between (5.6) thus obtained and the general expansion in (5.3) leads to the following restrictions

$$s_{ab} J_\mu^{(c)} = \bar{K}_\mu^{(c)} = 0 \quad s_b J_\mu^{(c)} = K_\mu^{(c)} = 0 \quad s_b s_{ab} J_\mu^{(c)} = L_\mu^{(c)} = L_{\mu 1}^{(c)} + L_{\mu 2}^{(c)} = 0. \quad (5.7)$$

A careful look at the expressions in (5.4) and (5.5) leads to the following solutions for the restrictions (5.7) in terms of the (anti-)ghost fields $(\bar{C})C$ and the matter fields ‡‡

$$\begin{aligned} \bar{f}_1 &= -e\bar{C}\phi & f_2 &= -eC\phi & \bar{f}_2^* &= +e\bar{C}\phi^* & f_1^* &= +eC\phi^* \\ b^* &= i e \phi^* [B - e\bar{C}C] & b &= -i e [B + e\bar{C}C] \phi. \end{aligned} \quad (5.8)$$

The explicit computation, with the above insertions, leads to the precise expression for $L_{\mu 1}^{(c)} = 2e(\partial_\mu B)(\phi^*\phi)$ which exactly cancels with the computed value of $L_{\mu 2}^{(c)}$, given by $L_{\mu 2}^{(c)} = -2e(\partial_\mu B)(\phi^*\phi)$. Rest of the conditions are also very beautifully satisfied which finally lead to the restriction on the supermanifold as $\tilde{J}_\mu^{(c)}(x, \theta, \bar{\theta}) = J_\mu^{(c)}(x)$. We wish to re-emphasize that this condition is *not* put by hand from outside. It is the inherent property of the theory itself. In other words, the off-shell nilpotent symmetries (2.6) for the matter fields are such that the supercurrent $\tilde{J}_\mu^{(c)}(x, \theta, \bar{\theta})$, even though expanded along $\hat{1}, \theta, \bar{\theta}$ and $\theta\bar{\theta}$ -directions of the six dimensional supermanifold, gets rid of its Grassmannian dependence and reduces to its local version $J_\mu^{(c)}(x)$ on the 4D manifold. Ultimately, the super expansion in (5.1), in the light of (5.8), becomes

$$\begin{aligned} \Phi(x, \theta, \bar{\theta}) &= \phi(x) + \theta (s_{ab}\phi(x)) + \bar{\theta} (s_b\phi(x)) + \theta \bar{\theta} (s_b s_{ab}\phi(x)) \\ \bar{\Phi}^*(x, \theta, \bar{\theta}) &= \phi^*(x) + \theta (s_{ab}\phi^*(x)) + \bar{\theta} (s_b\phi^*(x)) + \theta \bar{\theta} (s_b s_{ab}\phi^*(x)). \end{aligned} \quad (5.9)$$

It is clear that the analogue of (4.9) can be written for the interacting $U(1)$ gauge theory involving the complex scalar fields with the replacements $\Sigma(x) = \phi(x), \phi^*(x)$ and $\tilde{\Sigma}(x, \theta, \bar{\theta}) = \Phi(x, \theta, \bar{\theta}), \bar{\Phi}^*(x, \theta, \bar{\theta})$. In a similar fashion, the analogue of (4.10) is also valid for the system of complex scalar fields in interaction with the $U(1)$ gauge field A_μ .

6 Conclusions

In our present investigation, we have addressed the long-standing problem of the derivation of the off-shell nilpotent (anti-)BRST symmetry transformations for the matter fields, present in the interacting Abelian $U(1)$ gauge theories, in the framework of augmented

‡‡ It is interesting to note that for the condition $s_b J_\mu^{(c)} = K_\mu^{(c)} = 0$ to be satisfied (cf (5.7)), it is clear that the *odd* looking term $A_\mu(\phi^* f_2 + f_1^* \phi)$ in (5.4) should be zero on its own. This can be easily achieved if the fermionic secondary fields f_2 and f_1^* are proportional to the basic bosonic fields ϕ and ϕ^* respectively. To make the latter pair fermionic in nature, a smart guess is $f_2 \sim -C\phi, f_1^* \sim C\phi^*$. Exactly the same kind of argument is valid for $s_{ab} J_\mu^{(c)} = \bar{K}_\mu^{(c)} = 0$ which entails upon the secondary fields to be: $\bar{f}_1 \sim -\bar{C}\phi, \bar{f}_2^* \sim \bar{C}\phi^*$. The rest of the choices in (5.8) follow exactly the similar kind of logical arguments.

superfield formalism. The field theoretical examples that we have chosen are (i) the Dirac fields in interaction with the $U(1)$ gauge field A_μ , and (ii) the interacting Abelian 1-form gauge theory involving the complex scalar fields as the matter fields. As it turned out, for both the above cases of the interacting field theories, it is the requirement of the invariance of the conserved (super)currents, defined on the (super)manifolds, that is responsible for the derivation of the off-shell nilpotent (anti-)BRST transformations on the matter fields. There is a clear-cut distinction, however, between the mechanism of derivation of the above symmetries for the cases of (i) the Dirac fields, and (ii) the complex scalar fields. For the case of the interacting Abelian gauge theory with the Dirac fields, the horizontality condition (which is responsible for the derivation of the nilpotent symmetries for the gauge- and (anti-)ghost fields (cf section 3)), *does not* play any significant role in the derivation of the corresponding nilpotent (anti-)BRST symmetry transformations for the matter fields (see, eg, section 4). This is due to the fact that the matter supercurrent $\tilde{J}_\mu^{(d)}(x, \theta, \bar{\theta}) = \bar{\Psi}(x, \theta, \bar{\theta})\gamma_\mu\Psi(x, \theta, \bar{\theta})$ does not contain any superfields corresponding to the basic fields A_μ, C, \bar{C} . On the contrary, the horizontality condition does play a very crucial and decisive role in the derivation of the nilpotent (anti-)BRST symmetry transformations for the complex scalar fields. This is primarily because of the fact that the conserved supercurrent $\tilde{J}_\mu^{(c)}(x, \theta, \bar{\theta})$ (cf (5.2)) contains the superfield $B_\mu(x, \theta, \bar{\theta})$ corresponding to the gauge field $A_\mu(x)$. While computing the super expansion for the $\tilde{J}_\mu^{(c)}(x, \theta, \bar{\theta})$ along $\hat{1}, \theta, \bar{\theta}$ and $\theta\bar{\theta}$ -directions of the supermanifold, we do require the expansion in (3.6) for the superfield $B_\mu(x, \theta, \bar{\theta})$ which is derived after the restriction (i.e. $\tilde{F} = F$) due to the horizontality condition is imposed on the (super) curvature 2-forms.

On the face value, it appears very surprising that the off-shell nilpotent transformations for the gauge- and (anti-)ghost fields derived from the horizontality condition are consistent with and complementary to the nilpotent transformations for the matter fields derived from the requirement of the invariance of the conserved matter (super)currents of the theory. However, there is an explanation for this mutual consistency and complementarity between the two. In fact, the nilpotent (anti-)BRST transformations for the gauge fields (that involve the (anti-)ghost fields) are encoded in the curvature 2-form $F = dA$ for the Abelian $U(1)$ gauge theory as it remains invariant under the (anti-)BRST transformations $A \rightarrow A + d\bar{C}, A \rightarrow A + dC$. This is why when we demand $\tilde{F} = F$ on the six $(4+2)$ -dimensional supermanifold, we obtain the transformations on the gauge- and (anti-)ghost fields. To express the same thing in the physical language, we just demand that the classical physical (i.e. BRST invariant) fields \mathbf{E} and \mathbf{B} , in the superfield formulation, *should not* get any contribution from the Grassmann variables. The next physically important object in the interacting $U(1)$ gauge theory is the matter conserved current which plays a significant role in the interaction term $J_\mu^{(c,d)}A^\mu$ (where the matter conserved current couples to the gauge field). The conserved matter current is derived due to the global gauge invariance in the theory (Noether theorem). However, the interaction term owes its origin to the requirement of the *local* gauge invariance (gauge principle [41]). Thus, the outcome from

the requirement of the *invariance* of the matter (super)currents on the (super)manifolds is mutually consistent with and complementary to the local gauge (i.e. BRST) invariance of the curvature 2-form $F = dA$ as the principle of *local* gauge invariance is the common and connecting thread that runs through both of the above requirements.

In the present paper, we have concentrated only on the local, covariant, continuous and off-shell nilpotent (anti-)BRST symmetry transformations for the matter fields. However, for the Dirac fields in interaction with the $U(1)$ gauge field A_μ , it is already known that there exists a set of non-local, non-covariant, continuous and nilpotent (anti-)co-BRST symmetry transformations (see, eg, [34] for detailed references). Such kind of symmetries for the Dirac fields have also been found for the non-Abelian gauge theory where there is a coupling between the $SU(N)$ gauge field and the matter (Dirac) conserved current [42]. It would be interesting endeavour to extend our present work to include these non-local and non-covariant symmetry transformations. Furthermore, our present investigation can be generalized readily to the *interacting* non-Abelian gauge theory where the local, covariant, continuous and off-shell nilpotent (anti-)BRST transformations do exist for the non-Abelian gauge field, the (anti-)ghost fields and the Dirac fields. The derivation of the on-shell version of the above symmetries for the interacting (non-)Abelian gauge theories with matter fields is another direction that can be pursued in the future. These are some of the issues that are under investigation and our results will be reported elsewhere [43].

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